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Noise-induced multi-decrease and multi-increase of net voltage in Josephson junctions

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Abstract
In this paper, we investigate the net voltage in the Josephson junction subject to a thermal noise, a dc constant bias electric current and an ac time-periodic electric drive for the overdamped case and underdamped case simultaneously. It is shown that, by increasing the thermal noise strength, the net voltage can be decreased and increased several times as a function of the driving frequency. In addition, noise-enhanced stability is found for the net voltage of the junction.

1. Introduction

Nowadays, the net voltage and the dc current–voltage characteristics for the electrical transport in Josephson junctions (superconducting junctions) [1] subject to noise have become more and more interesting [2–12]. It has been shown that the asymmetric noise can produce a net voltage [2, 3] and dc voltage rectification [4]. We have found that the correlated symmetric noise (thermal noise and external perturbation noise) can also induce a net voltage that stems from the symmetry breaking induced by the correlation between the internal thermal fluctuation and the external perturbation [3, 5]. Zapata *et al* proposed a device with three Josephson junctions (a SQUID (superconducting quantum interference device) ratchet), threaded by a magnetic flux and driven by a thermal noise and a periodic signal [6], and studied the current–voltage characteristics for this device. We investigated the net voltage, the dc current–voltage characteristics and the mean first passage time (or the exit time, or escape time)[1] for the particles to escape over the fluctuating potential barrier for this device in the case of thermal fluctuation, together with the environmental perturbation [7]. In addition, in [8], we investigated the chaotic-noisy electrical transport in this device driven by thermal noise and oscillatory drive for the underdamped case (thermal-inertial ratchet). Sterck *et al* realized the device proposed by Zapata *et al* [6] and demonstrated the operation of those devices as very efficient rocking ratchets [9]. In our recent publication [10], we investigated the appearance of spatiotemporal noise and its effect on the electrical transport for this device. In a very recent paper [11], Kostur *et al* investigated the electrical transport in a Josephson junction driven by thermal noise, a constant bias force and a time-periodic signal (or oscillatory drive). They observed some anomalous behavior for the current (or flux) of the electrical transport as a function of the constant bias force, i.e. absolute negative conductance (ANC), negative differential conductance (NDC) and negative-valued nonlinear conductance (NNC) (the phenomena of ANC, NDC and NNC all belong to the category of negative mobility phenomenon [12]).

In this paper, we will study the net voltage [2] for a Josephson junction subject to thermal noise, a dc constant bias electric current and an ac time-periodic drive (or oscillatory signal), in the overdamped case and in the underdamped case simultaneously (the underdamped case is the one investigated by Kostur *et al* [11]), and will focus our investigation on the net voltage as a function of the oscillatory driving frequency and the thermal noise strength (Kostur *et al* [11] only considered the physical characteristics for the current as a function of the constant bias force for the underdamped case). It will show that the net voltage can be decreased and increased several times by increasing the thermal noise strength, at the peak positions and the well positions, respectively, as a function of the driving frequency. In addition, noise-enhanced stability will be found by us for the net voltage.

[1] For the Josephson junction, it has a periodic potential for the phase difference across the junction [1]. Here the escape time (or exit time) is the mean first passage time [18] for the particles to escape over one period of the potential of the Josephson junction [7, 28, 29].

2. Overdamped case

We carry out our study on the Josephson junction whose phase difference across the junction is a semiclassical variable and which can be adequately described by the 'resistively shunted junction' model [13, 14]. Then, the phase difference ϕ across the junction obeys the equation

$$I(t) = J_c \sin\phi + \frac{\hbar}{2eR}\frac{d\phi}{dt} + \frac{\hbar C}{2e}\frac{d^2\phi}{dt^2}, \tag{1}$$

where $I(t)$ is the total current through the junction, and R, C and J_c are the resistance, capacitance and critical current of the junction. If $(2e/\hbar)J_c R^2 C \ll 1$, we can neglect the capacitance C in equation (1) [2, 6, 7, 14]. Then, the evolution of the phase difference ϕ across the superconducting junction can be described by the equation [2, 6, 7, 14]

$$\frac{\hbar}{2eR}\frac{d\phi}{dt} + J_c \sin\phi = I(t). \tag{2}$$

After considering the thermal noise $\xi(t)$, and feeding an ac oscillatory drive current (time-periodic signal) $A\cos(\omega t)$ and a dc constant bias current I_0 to the junction, i.e. $I(t) = \xi(t) + A\cos(\omega t) + I_0$, equation (2) is

$$\frac{\hbar}{2eR}\frac{d\phi}{dt} + J_c \sin\phi = \xi(t) + A\cos(\omega t) + I_0, \tag{3}$$

where A and ω are the amplitude and frequency of the ac oscillatory drive, and $\xi(t)$ is the thermal Gaussian white noise with zero mean and correlation function $\langle\xi(t)\xi(\tau)\rangle = 2D\delta(t-\tau)$ (the noise strength $D = k_B T/R$, where k_B is the Boltzmann constant and T the temperature).

Below we transform equation (3) into a dimensionless form. Since the capacitance $C = 0$, we cannot use the Josephson plasma frequency $\omega_p = [2eJ_c/(\hbar C)]^{1/2}$ to introduce the timescale [11, 15]. But now we can use the characteristic frequency of the junction, $\omega_c = 2eJ_c R/\hbar$ [15]. So, here we introduce the timescale $\tau_0 = 1/\omega_c = \hbar/(2eRJ_c)$. After that, equation (3) can be transformed into the following dimensionless form:

$$\frac{d\phi}{dt'} = -\sin(\phi) + \eta(t') + a\cos(\omega' t') + i_0, \tag{4}$$

in which $t' = t/\tau_0$ with $\tau_0 = \hbar/(2eRJ_c)$, $\eta(t')$ is a dimensionless Gaussian white noise whose strength is $D' = 4ek_B T/(J_c\hbar)$, $a = A/J_c$, $\omega' = \tau_c\omega$ and $i_0 = I_0/J_c$.

The Fokker–Planck equation of equation (4) for the probability density $P(\phi, t')$ can be easily obtained. But we cannot get its analytical solution, even in the stationary state, since the detailed balance is broken and the probability flux is not zero (of course, we can solve the Fokker–Planck equation using numerical methods). Here, we carry out the numerical simulation directly using the Langevin equation (4). The numerical algorithm of equation (4) is [16]

$$\phi(t' + \Delta t') = \phi(t') + \tfrac{1}{2}(F_1 + F_2)\Delta t' + X(t', \Delta t'), \tag{5}$$

where $F_1 = -\sin(\phi) + a\cos(\omega' t') + i_0$, $F_2 = -\sin[\phi(t') + F_1\Delta t' + X(t', \Delta t')] + a\cos(\omega' t') + i_0$ and $X(t', \Delta t') = r(t')\sqrt{2D'\Delta t'}$, with $r(t')$ being a Gaussian random number of zero mean and variance 1.

We define the current J [8, 17] which is averaged over a sufficiently long time for the average velocity of the variable ϕ. Therefore, the current has two different averages. The first average is performed over ensembles of M (in the paper we take $M = 2000$) trajectories starting from different initial conditions. For every trajectory, we use a different Gaussian white noise. So, our first average is not only over the initial conditions but also over the realizations of the noise. For a fixed time t'_i, we can obtain the first average (i.e. the average velocity) $v_i = (1/M)\sum_{j=1}^{M} d\phi_j(t'_i)/dt'_i$. The second average is a time average (the time step is taken as $\Delta t' = 0.001$). In order to guarantee that the system is in the stationary state, the time average is taken after $t' = 1000$ (this average is taken from $t' = 1000$ to 2000). Then we have a discrete finite set of $N = 10^6$ different times t'_i. The current is $J = 1/N\sum_{i=1}^{N} v_i$.

Some explanations for the current J defined by us are given below. (1) The current J defined by us does not refer to the electrical current, but to the time average of the average velocity of ϕ (i.e. $J = \langle d\phi/dt\rangle_s$ in which $\langle\ \rangle_s$ means the average over the initial conditions, the noise and the time for the stationary state [8]). (2) For the net voltage $\langle V\rangle_s$, we know that $\langle V\rangle_s = (\hbar/2e)\langle d\phi/dt\rangle_s$ [2, 3, 8], so we can get $\langle V\rangle_s = (\hbar/2e)J$. (3) For the stationary probability current J_{spc} of equation (4), it satisfies $\langle d\phi/dt\rangle_s = LJ_{spc}$, where L is the spatial period of the system (see equation (11) of [19]), so for equation (4) we can get its stationary probability current $J_{spc} = J/(2\pi)$. In a word, the current J defined by us in the present paper can represent the net voltage of the junction or the stationary probability current of equation (4) (or equation (7)).

Based on the algorithm equation (5), in figure 1, we plot some results of our numerical simulations for J, which is proportional to the net voltage of the junction and the stationary probability current of equation (4), versus the natural logarithm (ln) of the frequency ω' of the time-periodic signal for different values of the noise strength D' ($D' = 0$, 0.001, 0.01 and 1), with the other parameters $i_0 = 0.3$ and $a = 2$. From this figure, we can see that, at the maximum positions of J as a function of the driving frequency, on increasing the noise's strength, the values of J can be decreased; while at the minimum positions, with the increase of the noise strength, the values of J can be increased. So, we call the phenomenon in figure 1 '*noise-induced multi-decrease and multi-increase of net voltage (or stationary probability current)*' (NMMNV or NMMSPC). Here, 'multi-' denotes that the net voltage (or the stationary probability current) can be decreased and increased several times (more than two times), which can be observed in figure 1. Now the nonlinearity of the net voltage of the junction (or stationary probability current of equation (4)) depending on the driving frequency leads to the appearance of several peaks and wells of the net voltage (or stationary probability current) versus the driving frequency.

Here, the noise-induced decrease of the net voltage (or the stationary probability current for equation (4)) corresponds to the prolongation of the escape time (or mean first passage time, or exit time) (see footnote 1) for the particles to escape over the fluctuating potential barrier with the increase in

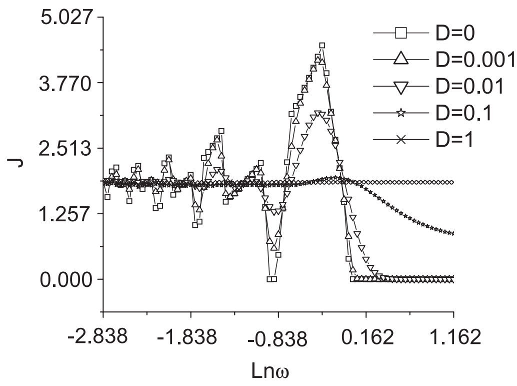

Figure 1. J as a function of the ln of the driving frequency ω' for equation (4), with different values of the noise's strength D' ($D' = 0$, 0.001, 0.01 and 1) and the other parameters $i_0 = 0.3$ and $a = 2$.

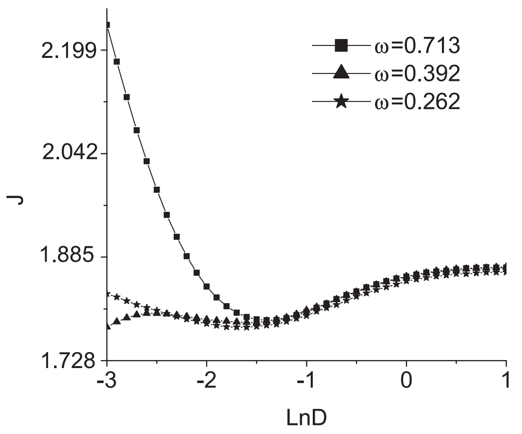

Figure 2. J versus the ln of the noise's strength D' for equation (4), with different values of the driving frequency ($\omega' = 0.713$, $\omega' = 0.392$ and $\omega' = 0.262$) and the other parameters $i_0 = 0.3$ and $a = 2$.

the noise strength, which is called 'noise-enhanced stability (NES)' [20–22]. For the NES phenomenon, a maximum for the mean first passage time (MFPT) versus the noise's strength can appear for most cases [20–22]. To find if a minimum is also present for J versus the noise's strength D' (the minimum for J versus the noise's strength corresponds to the maximum for the MFPT versus the noise's strength), in figure 2 we plot some results of J versus the ln of the noise's strength D' for different values of the driving frequency ($\omega' = 0.713$, $\omega' = 0.392$ and $\omega' = 0.262$) with the other parameters $i_0 = 0.3$ and $a = 2$. This figure shows that the net voltage (or the stationary probability current for equation (4)) versus the noise's strength has a 'well', which accords with the NES phenomenon that the escape time over the fluctuating potential barrier has a maximum as a function of the noise's strength. So, we call this phenomenon *NES for the stationary probability current* (NES-SPC). Moreover, from figure 2, we can see that, in the vicinity of the right-hand side of the well, on increasing the noise's strength, the net voltage (or the stationary probability current for equation (4)) can be increased; while in the vicinity of the left-hand side of the well, with an increase of the noise's strength, the net voltage (or the stationary probability current for equation (4)) can be decreased. So, figure 2 can clearly show the emergence of the decrease and increase of the net voltage (or the stationary probability current for equation (4)) induced by the thermal noise, even though the multi-decrease and multi-increase of the net voltage (or the stationary probability current for equation (4)) are absent in this figure.

Of course, in the absence of noise, no NMMNV (or NMMSPC) phenomenon or NES-SPC phenomenon appear. In addition, our study shows that the NMMNV (or NMMSPC) phenomenon and the NES-SPC phenomenon can only emerge for certain values of the periodic-driving frequency and the periodic-driving amplitude. Thus, the noise and the external periodic-oscillatory drive are ingredients for the appearance of the NMMNV (or NMMSPC) phenomenon and the NES-SPC phenomenon of our overdamped Josephson junction model (i.e. equation (3)). Up to now (to our knowledge), the NMMNV (or NMMSPC) phenomenon and the NES-SPC phenomenon reported by us in this paper are the first observation for the net voltage in a Josephson junction (or for the stationary probability current in a spatially periodic *symmetric* system). It remains to be studied whether they exist in the spatially periodic *asymmetric* systems driven by noise (our model for the Josephson junction belongs to the spatially periodic *symmetric* systems [23]).

3. Underdamped case

If the capacitance of the Josephson junction cannot be neglected, after feeding the junction with an ac oscillatory drive current $I_{ac} = A\cos(\omega t)$ and a dc constant bias force current $I_{dc} = F$, and considering the thermal noise $\xi(t)$ that is the same as the one in equation (3), from equation (1) one can get

$$\frac{\hbar C}{2e}\frac{d^2\phi}{dt^2} + \frac{\hbar}{2eR}\frac{d\phi}{dt} + J_c \sin\phi = A\cos(\omega t) + F + \xi(t). \quad (6)$$

In order to make a detailed investigation of equation (6), we transform it into a dimensionless form. Here we adopt the dimensionless form for equation (6) from [11] and [15]. In this case, the dimensionless form of equation (6) is

$$\frac{d^2\phi}{dt'^2} + \gamma\frac{d\phi}{dt'} = -\sin(\phi) + A_0\cos(\omega' t') + F_0 + \zeta(t'), \quad (7)$$

where $t' = t/\tau_0$ with $\tau_0 = 1/\omega_p = \sqrt{\hbar C/(2eJ_c)}$ (ω_p is the Josephson plasma frequency), $\gamma = \tau_0/\tau_r = \tau_0/RC$ (τ_r is the relaxation time), $A_0 = A/J_c$, $\omega' = \tau_0\omega$, $F_0 = F/J_c$ and $\zeta(t')$ is a dimensionless Gaussian white noise whose strength is $D_0 = 2k_B T/(RJ_c^2\tau_0)$.

The Fokker–Planck equation (FPE) for the probability density of equation (7) can be found easily. But we cannot get its analytical solution, since the probability flux is nonzero and the detailed balance is broken (of course, we can solve the FPE using numerical simulation). Here, we carry out our numerical simulation directly using the Langevin equation (7). According to [16], the numerical algorithm of equation (7) can

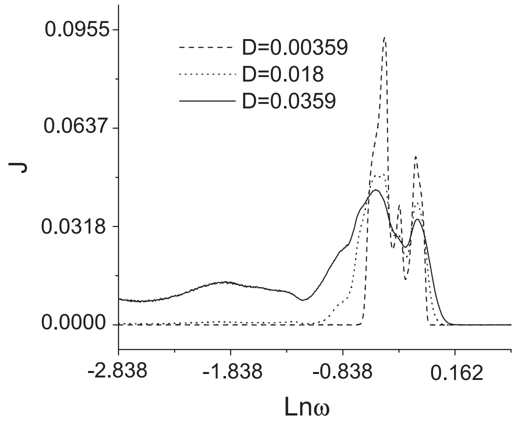

Figure 3. J versus the ln of the driving frequency ω' for equation (7), with different values of the noise's strength D_0 ($D_0 = 0.00359$, 0.018 and 0.0359) and the other parameters $\gamma = 0.143$, $F_0 = 0.0637$ and $A_0 = 0.668$.

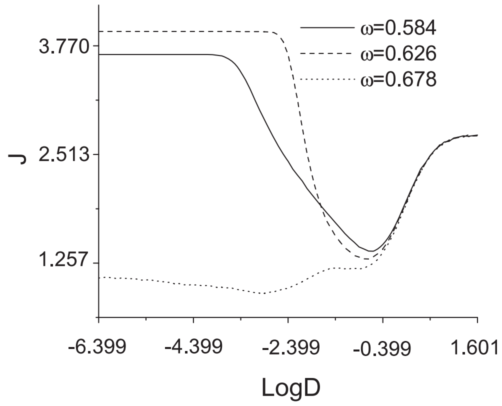

Figure 4. J versus the common logarithm (log) of the noise's strength D_0 for equation (7), with different values of the driving frequency ω' ($\omega' = 0.584$, $\omega' = 0.626$ and $\omega' = 0.678$) and the other parameters $\gamma = 0.143$, $F_0 = 0.0637$ and $A_0 = 0.668$.

be obtained:

$$\phi(t'+\Delta t') = \phi(t') + \tfrac{1}{2}(y(t') + F'')\Delta t',$$
$$y(t'+\Delta t') = y(t') + [-\gamma\tfrac{1}{2}(y(t') + F'') - \tfrac{1}{2}(F_1' + F_2'') + A_0\cos(\omega' t') + F_0]\Delta t' + X(t', \Delta t'), \qquad (8)$$

in which $y(t') = \mathrm{d}\phi(t')/\mathrm{d}t'$, $F'' = y(t') + [-\gamma y(t') - F_1' + A_0\cos(\omega' t') + F_0]\Delta t' + X(t', \Delta t')$, $F_1' = \sin(\phi(t'))$, $F_2'' = \sin(\phi(t') + y(t')\Delta t')$ and $X(t', \Delta t') = r(t')\sqrt{2D_0\Delta t'}$, with $r(t')$ being a Gaussian random number of zero mean and variance 1.

Based on the numerical algorithm equation (8), in figure 3 we plot some results of our numerical simulations for J (which is proportional to the net voltage of the junction and the stationary probability current of equation (7)) as a function of the ln of the driving frequency ω' for different values of the noise's strength D_0 ($D_0 = 0.003\,59$, 0.018 and 0.0359), with the other parameters $\gamma = 0.143$, $F_0 = 0.0637$ and $A_0 = 0.668$. From figure 3, we can observe that, in the vicinity of the maxima for J versus the driving frequency, on increasing the noise's strength, the net voltage (or the stationary probability current of equation (7)) can be decreased, which accords with the NES phenomenon for the escape time of the particles to escape over the fluctuating potential barrier (here that the net voltage (or the stationary probability current of equation (7)) is decreased corresponds to that the escape time is prolonged); while in the vicinity of the minima for J versus the driving frequency, with the increase of the noise strength, the net voltage (or the stationary probability current of equation (7)) can be increased. Thus, the NMMNV (or NMMSPC) phenomenon found by us in section 2 is also present for the underdamped case of the Josephson junction.

Moreover, one knows that, for the NES phenomenon, a maximum for the escape time as a function of the noise's strength can appear in most cases [20–22]. To find this phenomenon for the net voltage (or the stationary probability current of equation (7)) as a function of the noise's strength, in figure 4, we depict some curves of J versus the common logarithm (log) of the noise's strength D_0 for different values of the driving frequency ω' ($\omega' = 0.584$, $\omega' = 0.626$ and $\omega' = 0.678$) with the other parameters $\gamma = 0.143$, $F_0 = 0.0637$ and $A_0 = 0.668$. This figure clearly shows the emergence of a minimum for the net voltage (or the stationary probability current of equation (7)) as a function of the noise's strength (the minimum for J corresponds to the maximum for the escape time). So, the NES-SPC phenomenon is also present for the underdamped case. In addition, in the present paper, what we are more interested in is the decrease and increase of the net voltage induced by the thermal noise. From figure 4, we can observe that, on the left-hand side of the well, with the increase of the noise strength, there are some certain regions where the net voltage can be decreased; while on the right-hand side of the well, with increasing the noise strength, the net voltage can be increased. So, figure 4 can be a clear representation for the appearance of the decrease and increase of the net voltage induced by the thermal noise. Our further study shows that, as in section 2 for the overdamped case of the Josephson junction, for the underdamped case, the oscillatory drive and the thermal noise are the ingredients for the emergence of the NMMNV (or NMMSPC) phenomenon and the NES-PC phenomenon.

4. Conclusion and discussion

In conclusion, in this paper, we have reported a phenomenon of the noise-induced multi-decrease and multi-increase of the net voltage and a phenomenon of noise-enhanced stability for the stationary probability current (or net voltage) in the Josephson junction for the overdamped case and the underdamped case simultaneously. The net voltage can be decreased and increased several times, as a function of the driving frequency, by increasing the thermal noise's strength at the peak positions and the well positions, respectively. The net voltage can represent a minimum as a function of the thermal noise's strength. Although our results were obtained by investigating the net voltage in the superconducting junction, they can be applied to systems whose differential equations satisfy equation (3) or (6) (or equation (4) or (7)). In addition, the results found by us for equations (4) and (7) belong to the transport of the particles (or para-particles) caused by the symmetry breaking in the spatially periodic systems [23], which has recently attracted a great deal of attention in a variety of contexts [23–27]. So, it remains to be studied whether the NMMSPC phenomenon and the NES-SPC phenomenon for the flux of the particles (or para-particles) exist in the other spatially periodic systems, such as the transport of cold atoms [24, 25], the transport of solitons in Bose–Einstein condensates [26], the transport of biological and artificial molecular motors [27], and so on.

In addition, we have noted that the NES phenomenon of the escape time for particles to escape over the fluctuating potential barrier in a Josephson junction has been investigated in [28] for the overdamped case and recently studied in [29] for the underdamped case. In the present paper, we have found the NES-SPC phenomenon for the net voltage in a Josephson junction.

Finally, it is worthwhile mentioning that, for the numerical simulations of equations (4) and (7), using the RKII (stochastic Runge–Kutta II) method [16] and using the Euler method, we can get the same results (i.e. figures 1–4 almost remain unchanged, especially for the physical meanings). In addition, it should be stressed that our results (i.e. the NMMNV phenomenon and the NES-SPC phenomenon) are only physically theoretical, and they remain to be found experimentally.

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